Walking Control for Feasibility at Limit of Kinematics Based on Virtual Leader-Follower

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1. Introduction

Control of biped robots is still a challenge as they are highly complex systems. One of the difficulties is due to nonlinearities in the dynamics of the robot. In the legged locomotion, a widely used strategy to simplify such complex dynamics is to model the robot as an inverted pendulum, where the whole body dynamics are reduced to the motion of its center of mass (COM) and the concentration of all reaction force from the ground in a single point known as Zero Moment Point or ZMP[1], where the horizontal moment around it is zero. The motion of the COM and ZMP are then mapped into the configuration space of the robot, i.e., the joints. Many works based on the COM-ZMP model showed its use for locomotion[2][3][4].

With a simplified model, information regarding kinematics constraints of the robot is not taken into account in the controller. When the referential COM is located outside the reachable range (workspace) of the robot, there is a mismatch between robot and model (**Fig.**1), and the system may reach a singular point, which may create ill-condition in terms of computation. Some works accomplished stable control even at singular configuration in case of manipulators[5][6], but they solved the problem for specific end-effector trajectories, mechanisms and/or type of singularity.

A conservative approach avoids the singular points by a certain margin. This restrains the robot motion, and thus, reduces the mobility range. This can be typically observed in bended-knee walks of many robots. Consequently, there is great care to design restraints for the motion and a detailed knowledge of the robot structure is required which is a great burden for its design. Another approach is to try to control the robot near or even at the singular points by the use of partial knowledge of the workspace and its explicit implementation as constraints on the controller[7].

Due to these facts, the present work aims to fill the gap between walking motion control based on the reduced model and kinematics constraints in the robot without any particular care about the robot structure and its workspace. Two main issues of the control on the boundaries of the workspace are ill-posed computation at singular configurations and the divergence of the controller output when the robot reaches the limit of kinematics as this kind of configuration usually leads to the loss of controllability.

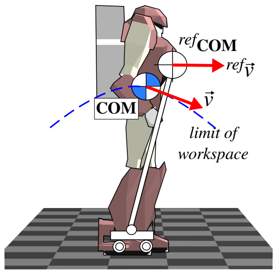

Fig.1 Mismatch between robot and COM-ZMP model, with model and robot velocities, ${}^{ref}v$ and v, respectively.

In the previous work[8], the authors could accomplish stable motions even at the limits of the robot kinematics by a robust prioritized IK solver[9] and by bounding the reference with a reset of system integrator. However, this bounding method assumes that the COM position exactly tracks the reference while it is in workspace. This work proposes a use of Virtual Leader-Follower (VLF) method[10] applied to the COM-ZMP model and the actual robot, which restricts the controller when robot reaches the limits of workspace and relaxes the above assumption.

2. COM control at limits of kinematics

2·1 COM-ZMP model

In this section, the robot model with simplified dynamics and its controller will be reviewed, as well as the main characteristics of the inverse kinematics solver and the system behavior at the boundary of workspace.

The equation of motion of the COM-ZMP model is defined as

$$\ddot{x}_G = \zeta^2 (x_G - x_Z) \tag{1}$$

$$\ddot{y}_G = \zeta^2 (y_G - y_Z) \tag{2}$$

$$\ddot{z}_G = \frac{f_z}{m} - g \tag{3}$$

$$\zeta^2 = \frac{\ddot{z}_G + g}{z_G - z_Z} = \frac{f_z}{m(z_G - z_Z)} \tag{4}$$

where $\boldsymbol{p}_G = [x_G \ y_G \ z_G]^{\mathrm{T}}$ is the COM position coordinates, m is its whole mass, $g = 9.8\mathrm{m/s}^2$ is the gravitational acceleration, f_z is the vertical force and $\boldsymbol{p}_Z = [x_Z \ y_Z \ z_Z]^{\mathrm{T}}$ is the ZMP coordinates, considered with $z_Z = 0$ hereafter. As the vertical force is gained as a reaction from the ground and ZMP must lie within the support region $\boldsymbol{S}$, the dynamical constraints are given by

$$\boldsymbol{p}_Z \in \boldsymbol{S}, \tag{5}$$

$$f_z \geq 0. \tag{6}$$

As ZMP and the vertical force are restricted by **Eqs.**(5) and (6), they are are chosen as the input variables of the system to manipulate the COM and are defined, for example, by the following feedback rules:

$$\tilde{\boldsymbol{p}}_Z = {}^d\boldsymbol{p} + \boldsymbol{K}_p \Delta \boldsymbol{p}_G + \boldsymbol{C}_p \Delta \dot{\boldsymbol{p}}_G \tag{7}$$

$$\tilde{f}_z = mg + k_z \Delta z_G + c_z \Delta \dot{z}_G, \tag{8}$$

where $\Delta(*)_G = {}^d(*)_G - (*)_G$ and $\tilde{z}_Z = 0$ by definition. The above rules are restricted to ensure the dynamical constraints (5) and (6) by:

$$\boldsymbol{p}_Z = \arg \min_{\boldsymbol{s}_Z \in \boldsymbol{S}} ||\boldsymbol{s}_Z - \tilde{\boldsymbol{p}}_Z|| \tag{9}$$

$$f_z = \begin{cases} \tilde{f}_z & (\tilde{f}_z > 0) \\ 0 & (\text{otherwise}). \end{cases} \tag{10}$$

The referential COM is obtained by integrating **Eqs.**(1), (2) and (3) . Together with other referential positions of body parts such as feet, they are applied to an inverse kinematics solver, which gives the desired joint angles that, in turn, are used as references for the joint servo control of the robot.

If the servos can provide enough torques such that in an infinitesimal time joint angles can reach their referential angles, and with no slipping of the feet, the robot can track its referential COM movement as long as they are inside the workspace.

2·2 Robust prioritized inverse kinematics[9]

While tasks at the limits of kinematics in general imply reaching a singular configuration and consequently generate ill-posedness, many inverse kinematics solvers can handle the indeterminancy of such configuration. Sugihara[9] proposed a method that is robust against singularities and always find a solution, even if it is physically unfeasible. Given certain task references, the solver finds a solution which minimizes a weighted Euclidean norm of the errors between references and robot parts. When high priority is given to the determined reference, the solver strictly satisfies it and then minimizes the low-priority references distances.

For the walking motion when the COM reference goes outside the reachable workspace, if priority is not given to the feet, the solution does not satisfy dynamical constraints as the feet lose contact with the ground. On the other hand, prioritization of feet contact increases the error of COM.

2·3 Limits of kinematics as system saturation

Provided that sufficient torque is always supplied, the robot COM position tracks the COM reference

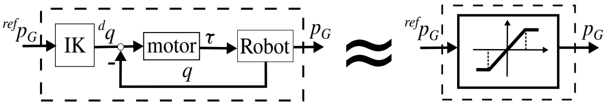

Fig.2 Representation of IK and robot as an equivalent saturation under sufficient torque assumption.

from the model as long as it is inside the workspace; otherwise, there is a mismatch between them. Such behavior can be regarded as a saturation function and upper and lower limits depend on the robot configuration as shown in **Fig.**2, where the robot COM position is given by:

$$\boldsymbol{p}_G = \begin{cases} \overline{\boldsymbol{p}}_G(\boldsymbol{q}, \boldsymbol{p}) & ({}^{ref}\boldsymbol{p}_G \notin \boldsymbol{W}) \\ {}^{ref}\boldsymbol{p}_G & ({}^{ref}\boldsymbol{p}_G \in \boldsymbol{W}) \end{cases}, \tag{11}$$

where ${}^{ref}\boldsymbol{p}_G$ and $\boldsymbol{p}_G$ are the referential and actual COM positions, respectively, $\boldsymbol{W}$ is the workspace of COM and $\overline{\boldsymbol{p}}_G(\boldsymbol{q}, \boldsymbol{p})$ is the boundary of workspace, which is a function of the robot joint angles $\boldsymbol{q}$ and the contact point $\boldsymbol{q}_0$. At the limits of kinematics the controller loses controllability and works in open-loop manner. The integral term of the system causes wind up of the output, and hence, it diverges. As it was experimentally demonstrated[8], large errors at low-priorities may overcome the effects of high-priorities on orientations and the system becomes unstable.

3. Virtual Leader-Follower

The basic idea is that the desired COM serves as the leader and the actual state tracks it as the follower. The dynamics of these two points are coupled by a virtual attraction force between them. The dynamics of the leader is defined by a driving force and the attraction force, while that of the follower by the attraction force. Both the points move at the same time. In other studies[11][12], the follower moves only due to the attraction force, and thus the leader must move first. Therefore the follower has a lag behind the leader. This lag is undesirable as the robot dynamics should be the same with the COM-ZMP model if the reference is reachable. Instead, the model dynamics is restrained by the attraction force such that:

$${}^{ref}\ddot{\boldsymbol{p}} = \xi^2({}^{ref}\boldsymbol{p} - {}^d\boldsymbol{p}_Z) - \boldsymbol{u}_t, \tag{12}$$

where $\boldsymbol{u}_t$ is the virtual attraction force, which here is considered as a spring-damper in the following form:

$$\boldsymbol{u}_t = \boldsymbol{K}({}^{ref}\boldsymbol{p}_G - \boldsymbol{p}_G) + \boldsymbol{C}({}^{ref}\dot{\boldsymbol{p}}_G - \dot{\boldsymbol{p}}_G) \tag{13}$$

and the robot will track the reference given by the model.

As the gap between the reference and the robot becomes larger, the attraction force also increases, which restrained the referential acceleration and, consequently, it avoids divergence of the controller.

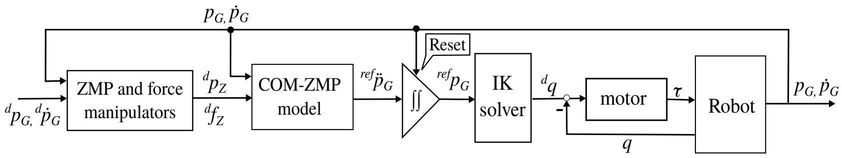

Fig.3 Previous serial system structure with COM-ZMP model and bounding of integrator

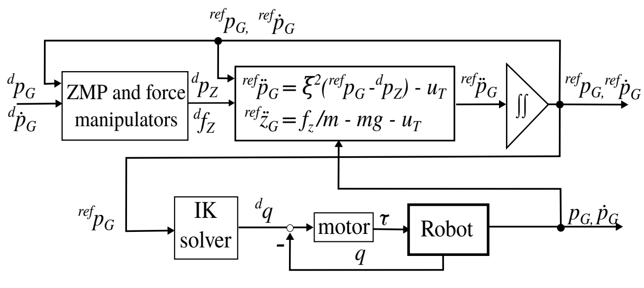

Fig.4 Proposed parallel system structure with COM-ZMP model and virtual leader follower.

The previous system relied on a serial control structure (**Fig.**3), while the proposed complete control system with the VLF method has a parallel structure as shown in **Fig.**4.

4. Simulation and Discussion

Simulations were performed in order to test the proposed method with a miniature humanoid robot[13] as the model, with total height of 58cm and COM height of 29.41cm when it stands upright. A forward walking control[3] was simulated with a desired velocity $^dv_x = 0.1$m/s and the referential height set initially with $^dz = 0.26$m, which was changed to $^dz = 0.3$m($t = 2$s), $^dz = 0.2$m($t = 6$s) and $^dz = 0.26$m ($t = 8$s).

Priorities are given to the feet poses and the parameters of the virtual leader follower were set empirically as $\boldsymbol{K} = diag\{300, 300, 300\}$, $\boldsymbol{C} = diag\{50, 50, 50\}$. The controller parameters were set according to Atsuta *et al.*[3]. **Fig.**5 shows snapshots of the simulated motion and **Fig.**6 shows profiles of the commanded COM, position of ZMP, robot COM and feet positions, and angles of the knee joints. As it can be seen in **Fig.**6(c), even with commanded height set outside the upper and lower limits of the COM height, the robot reaches such boundaries, while keeping a stable walking motion. Because of the virtual attraction force, the reference COM position is restrained by the robot position, i.e., even though the feedback loop is broken due to kinematic limits of the robot, the model does not lose trackability of the system.

The middle snapshot in **Fig.**5 shows the robot with fully stretched knee while **Fig.**6(d) shows the profiles of angles of the left and right knees during motion, where the full stretch of legs is characterized by the angles values reaching 0 degree.

Fig.7 shows the results in z-axis and xy-plane of simulations for backward, sideward and turning motions, where the commanded height was set above the reachable upper limit of COM. As it can be observed, similar results were obtained, i.e., stable motions were attained even at the limits of kinematics, demostrating applicability of the method.

5. Conclusion

This work tackles the issues of mobility of the robot and mismatch with the simplified model due to kinematics constrainted.

By the use of virtual leader-follower applied to the COM-ZMP model, even when the robot reaches the boundary of workspace and the controller loses controllability, a stable motion is achieved at the limits of kinematics. Moreover, the perfect tracking of COM within the workspace, which causes another trouble due to estimation error is no longer required. It is noteworthy that the proposed method was applied to an existing simple-model walking controller, which suggests its applicability to different controllers and it is unconcerned with trajectories and robot structure.

It was hypothesized that IK solver with robot body are equivalent to a saturation, but as the method was validated in a simplified dynamics environment, the issues of self-collision and constrained torques were not considered in the present work and topics of future research.

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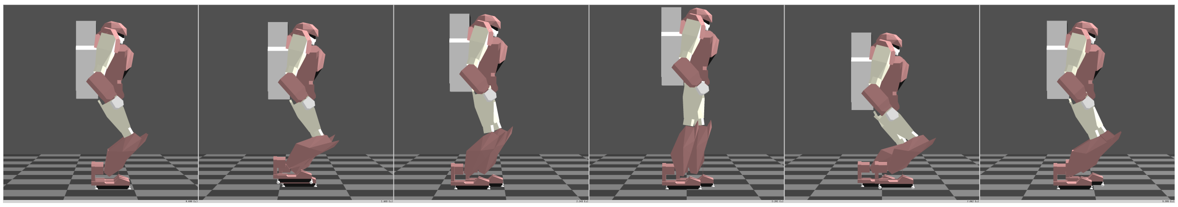
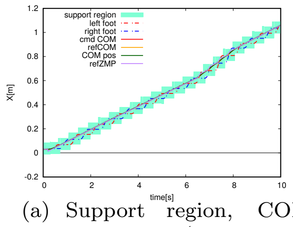
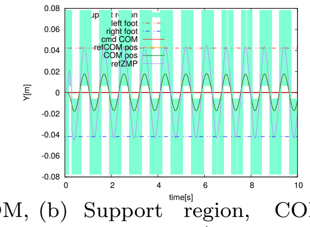


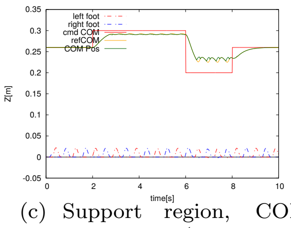
Fig.5 Comparative snapshots of forward motion with virtual leader follower method implemented



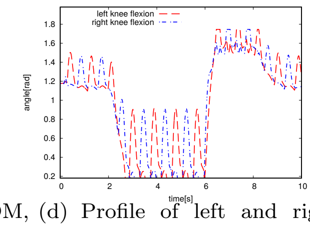
(a) Support region, COM, ZMP and feet (x-axis direction)



(b) Support region, COM, ZMP and feet (y-axis direction)

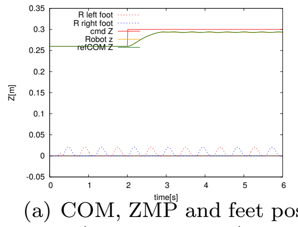


(c) Support region, COM, ZMP and feet (z-axis direction)

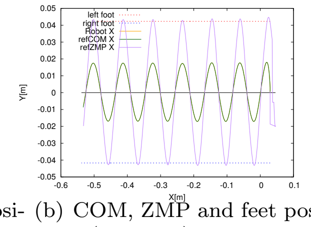


(d) Profile of left and right knees flexion angles

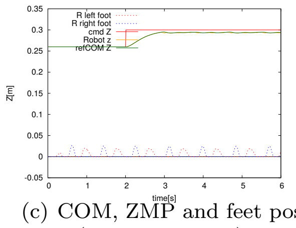
Fig.6 COM reference, position, ZMP and knee angles profile for forward walking



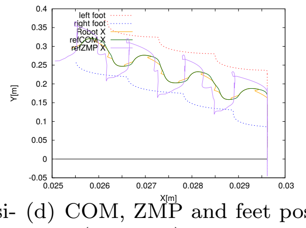
(a) COM, ZMP and feet positions (z-axis direction)



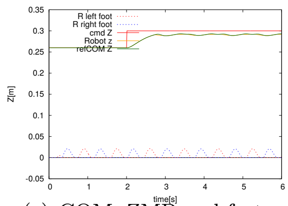
(b) COM, ZMP and feet positions (xy-plane)



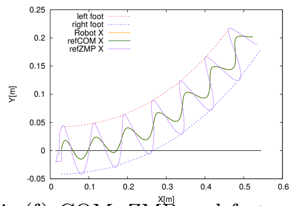
(c) COM, ZMP and feet positions (z-axis direction)



(d) COM, ZMP and feet positions (xy-plane)



(e) COM, ZMP and feet positions (z-axis direction)



(f) COM, ZMP and feet positions (xy-plane)

Fig.7 Motion of COM, ZMP and feet for (a)(b)backward, (c)(d)sidewards and (e)(f)turning motions for commanded height outside workspace

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