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Dynamics Morphing between Standing and Repetitive Hopping of Biped Robots

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Abstract— **This paper presents a repetitive hopping control of biped robots, which has the following properties: 1) It can initiate or cease hopping at any time, 2) The peak altitude can be freely altered during hopping, and 3) The impact at touchdown is suppressed. Hopping motions accompany frequent changes of contact conditions with the environment and lead the discontinuity of dynamics they are governed, therefore, it is difficult to achieve the motion in a way without relying any passive mechanisms such as springs and dampers. Three controllers, that is, (I) a regulator to stabilize, (II) a controller to converge a desired velocity when lifting off the ground and (III) a controller to absorb the landing impact are unified by the global scheme to transit each other depending on a commanded value and the state of the robot.**

I. Introduction

From a standpoint of locomotion capability particularly on complex terrain, legged robots are superior to wheeled or tracked ones since they can step over obstacles and climb stairs. To further enhance the mobility of legged robots, motions including flight phases such as jumping and running should be realized. Particularly, stabilization of biped robots when they are subjected to external forces may require some small hoppings. Since humanoid robots are expected to perform precise positioning as well as dynamic motions, it is undesirable for them to sacrifice controllability through the use of machine elements to absorb impacts such as springs and dampers more than necessity, thus, it is worth developing control theory to realize such motions.

Experimental research on a hopping mechanism may have started with Matsuoka [1], who analyzed a two-link model consisted with a massless leg and an upper body, and constructed a one-legged hopping machine constrained to move on an inclined plane. Raibert [2] and his colleagues presented stable running motion of a hydraulically actuated robot with decoupled control laws. Using modified Raibert's controller, Gregorio et al. [3] realized an energy-efficient hopping with electrical actuators and a spring-supported system. Hyon et al. [4] developed a hopping robot imitating the structure of a hind limb of a dog. These types of robots focused on development of a hopping machine and its control. However, they are not suitable for daily tasks of humanoid robots. Hopping motions themselves can be understood as variable constraint systems from a control theory point of view, that is, mechanical constraints vary as the contact condition changes [5]. Many researchers have treated the discontinuously changing dynamics of hopping or running motions with biped robots. Hirano et al. [6] suggested a cyclic jumping control of biped robots using the adaptive impedance control. Kajita et al. [7] proposed an offline running pattern planning method. However, these methods have some limitations, such as the difficulty of changing the robot's control strategy in real time. Mita et al. [8] proposed a control methodology called a variable constraint control, which can be applied to underactuated systems with some constraints, and demonstrated mono-leg, biped and quadruped running. Nagasaka et al. [9] put forward a method to plan motion trajectories of the whole body which satisfy dynamical constraints. These two techniques require accurate dynamical model of the robot, therefore, computational cost is high.

Online trajectory-based approaches are also proposed by Sugihara et al. [10], Tajima et al. [11], Ugurlu et al. [12] and so on. These methods are useful for a sophisticated motion generation, however, dynamic motions such as jumping and running require robustness against unknown disturbances rather than preciseness of trajectory tracking. Sugihara [13]–[15] proposed a framework called *dynamics morphing*, in which manipulation of reaction forces and contact conditions with environment are designed as a comprehensive structure of the dynamical system comprising the robot and the controller, and various motions such as standing [13], stepping [14] and longitudinal walking [15] are integrated seamlessly. This transition is realized by a continuous or a discontinuous morphing of the controller. In this paper, after the idea of dynamics morphing, we will realize standing and repetitive hopping of biped robots by designing the controller based on the perspective what structure the whole system should have as the dynamical system. Specifically, we will apply a lateral nonlinear oscillation of the center of mass (COM) that was needed for the stepping motion [14] for a cyclic motion in vertical direction, and propose a control method to transit freely between standing and repetitive hopping. This is achieved by the following controllers:

- (I) a regulator to stabilize a robot at a referential position
- (II) a controller to converge asymptotically to arbitrary velocity for reaching a desired peak altitude at the moment of liftoff
- (III) a controller to absorb the impulsive shock exerted on touchdown,

and we propose a global control scheme to transit these controllers (I)(II)(III) depending on a commanded value and the state of the robot.

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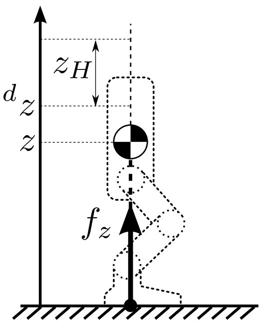


Fig. 1. Mass-concentrated model of a biped robot

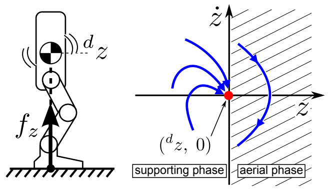


Fig. 2. Stabilization of the robot at a referential position.

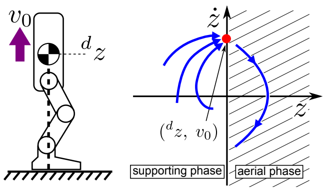


Fig. 3. A moment that the robot lifts off at $\dot{z} = v_0 = \sqrt{2gz_H}$

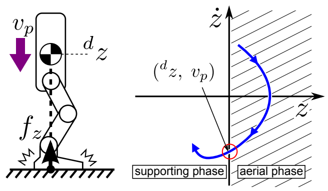


Fig. 4. A moment the robot touches on the ground

II. Model and Controllers for Different Objectives

A. Mass-concentrated model and equation of motion

In general, a biped robot is consisted with many links and joints, resulting in the complex formulation of the equation of motion. For the sake of simplicity, we consider a biped robot model depicted as Fig. 1, where the mass of the whole body is concentrated at COM. Let us assign z axis along with the vertical direction, and denote the position of COM as z and the desired hopping height as z_H. We postulate that COM movement is constrained in the vertical, the moment about COM is small enough to be ignored in the supporting and aerial phase and the robot lifts off or touch on the ground at $z = {}^d z$. Then, the following simplified equation of motion is obtained:

$$\ddot{z} = \frac{f_z}{m} - g, \tag{1}$$

where f_z is the vertical ground reaction force, m is the robot mass and $g = 9.8$[m/s^2] is the acceleration due to gravity. Suppose that the robot can always produce the whole of joint torques enough to generate a certain magnitude of the vertical reaction force. Eq. (1) indicates that the position of COM can be controlled via manipulation of the ground reaction force f_z. Since any pulling forces cannot be generated at any contact points, the following constraint condition exists:

$$f_z \geq 0. \tag{2}$$

In particular, $f_z = 0$ means that the robot has no contact points.

B. Regulator to stabilize at a referential position

By formulating the vertical ground reaction force for different objectives, desired motions of the biped robot will be achieved. First of all, let us design a regulator to stabilize at a referential position ${}^d z$. We define the vertical reaction force as

$$\tilde{f}_z^{\mathrm{I}} = k_1(z - {}^d z) + k_2\dot{z} + mg, \tag{3}$$

where k_1 and k_2 are feedback gains. The desired vertical reaction force f_z is defined as

$$f_z = \begin{cases} \tilde{f}_z^{\mathrm{I}} & (\mathrm{S^I1} : \tilde{f}_z^{\mathrm{I}} > 0 \quad \text{and} \quad z \leq {}^d z) \\ 0 & (\mathrm{S^I2} : \tilde{f}_z^{\mathrm{I}} \leq 0 \quad \text{or} \quad z > {}^d z) \end{cases} \tag{4}$$

with the constraint condition (2) taken into account. If the robot actually obtain the desired vertical reaction force from the ground, the motion of COM conforms to the following piecewise autonomous system:

$$\ddot{z} = \begin{cases} \dfrac{k_1}{m}(z - {}^d z) + \dfrac{k_2}{m}\dot{z} & (\mathrm{S^I1}) \\ -g & (\mathrm{S^I2}) \end{cases}. \tag{5}$$

Suppose the desired poles in (S$^{\mathrm{I}}$1) are assigned to $-\xi q_1$ and $-\xi q_2$, the feedback gains k_1 and k_2 are defined as

$$k_1 = -m\xi^2 q_1 q_2, \quad k_2 = -m\xi(q_1 + q_2), \tag{6}$$

where $\xi \equiv \sqrt{g/{}^d z}$ is a parameter to nondimentionalize q_1 and q_2.

Dynamical behaviors of the piecewise linear system dominated by Eq.(5) can be visualized by solution curves in phase space. Fig.5(a) shows a phase portrait for ${}^d z = 0.3$[m], $q_1 = 2.0$ and $q_2 = 3.0$. From the figure we can observe that COM state $(z,\ \dot{z})$ stably converges to the reference near $(z,\ \dot{z}) \cong ({}^d z,\ 0.0)$ and $z < {}^d z$. Tracking the solution curves in Fig. 5(a) along time evolution, we can find that they have a sharp bend at $z = 0.3$[m]. It should be noted that this results in exertion of an impulse shock at the moment of touchdown.

C. Attractor to converge to arbitrary velocity at liftoff

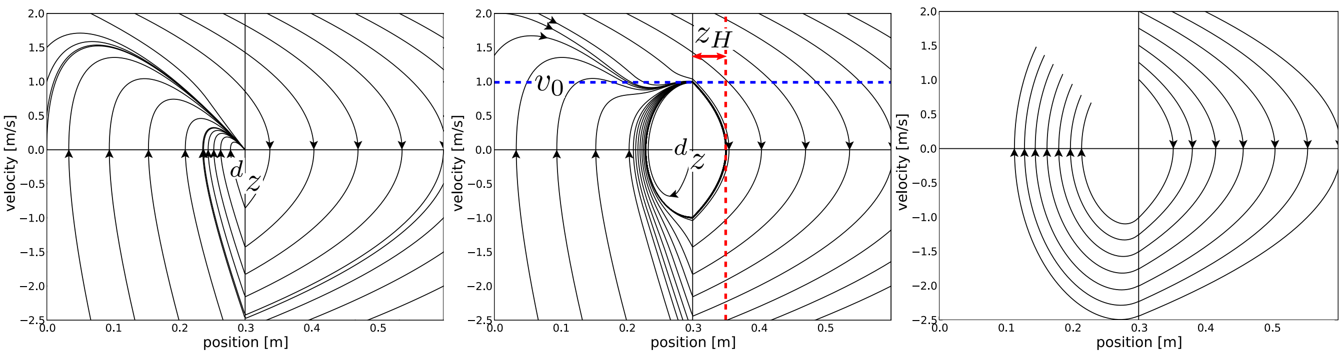
In order to lift off the ground at $z = {}^d z$ and jump z_H height, the velocity of COM just before lifting off should be $\dot{z} = v_0 = \sqrt{2gz_H}$. Therefore, at the moment to lift off, i.e. $z = {}^d z$, COM velocity needs to be asymptotically converged to v_0. As a linear feedback regulator can never achieve such characteristics, it is required to design a nonlinear feedback controller which works as a nonlinear oscillator. Applying a self-excited oscillator proposed by Sugihara [16], which is used for lateral oscillation of COM to step, we redefine the desired reaction force instead of (4) as

$$f_z = \begin{cases} \tilde{f}_z^{\mathrm{II}} & (\mathrm{S^{II}1} : \tilde{f}_z^{\mathrm{II}} > 0 \quad \text{and} \quad z \leq {}^d z) \\ 0 & (\mathrm{S^{II}2} : \tilde{f}_z^{\mathrm{II}} \leq 0 \quad \text{or} \quad z > {}^d z) \end{cases} \tag{7}$$

$$\tilde{f}_z^{\mathrm{II}} = -m\xi^2 q_1 q_2(z - {}^d z) - m\xi(q_1 + q_2)c^{\mathrm{II}}(z, \dot{z})\dot{z} + mg, \tag{8}$$

where

$$c^{\mathrm{II}}(z, \dot{z}) \equiv 1 - \rho \exp\left[1 - \gamma\left\{(z - {}^d z)^2 + \frac{\dot{z}^2}{\xi^2 q_1 q_2}\right\}\right], \tag{9}$$



(a) Regulator (b) Controller to hop repetitively (c) Controller to absorb the impact

Fig. 5. Phase portraits of the system in the case of using each controllers. (a): The regulator defined by Eq. (4) is applied. The point $(^d z, 0)$ on the phase plane should be stable equilibrium. (b): The controller defined by Eq. (7) is applied. The state on the phase plane should converge to $(^d z, v_0)$ for stable hopping. (c): The controller defined by Eq. (15) is applied. The state on the phase plane should traverse smoothly.

$\rho(\geq 0)$ and $\gamma(\geq 0)$ are adjustable controller parameters, setting maneuvers of which will be described later. It should be kept in mind that Eq. (7) coincides with the regulator (4) when $\rho = 0$. The nonlinear term $c^{\mathrm{II}}(z, \dot{z})$ has the effect to arise the self-excited oscillation to COM. It emerges the following stable limit cycle on the phase space in state ($\mathrm{S}^{\mathrm{II}}2$) when $\rho = 1$,

$$(z - {}^d z)^2 + \frac{\dot{z}^2}{\xi^2 q_1 q_2} = \frac{1}{\gamma}. \tag{10}$$

According to Eq. (10), γ determines the size of the limit cycle. If COM behavior conforms to the limit cycle, COM velocity will be $\xi\sqrt{\frac{q_1 q_2}{\gamma}}$ when reaching $z = {}^d z$ from the supporting phase $z < {}^d z$. To match the velocity with v_0, γ must be set as

$$\gamma = \frac{\xi^2 q_1 q_2}{v_0^2} \equiv \gamma_0. \tag{11}$$

When the control input (7) is given and $\gamma = \gamma_0$, the dynamics of COM depicted as the following piecewise autonomous system:

$$\ddot{z} = \begin{cases} -\xi(q_1 + q_2)c^{\mathrm{II}}(z, \dot{z})\dot{z} - \xi^2 q_1 q_2 (z - {}^d z) & (\mathrm{S}^{\mathrm{II}}1) \\ -g & (\mathrm{S}^{\mathrm{II}}2) \end{cases} \tag{12}$$

Solution curves of Eq. (12) for ${}^d z = 0.3$, $z_H = 0.05$ $q_1 = 2.0$, $q_2 = 3.0$ and $\rho = 1.0$ are shown in Fig. 5(b). The vertical red dotted line is plotted as $z = {}^d z + z_H = 0.035$[m] and the horizontal blue dotted line as $\dot{z} = v_0 \approx 0.99$[m/s]. Tracking the solution curves along time evolution, we can find that COM velocity asymptotically converges to $\dot{z} = v_0$ when reaching $z = {}^d z$ from the supporting phase $z < {}^d z$. After that, the robot changes to the flight phase and COM reaches $z = {}^d z + z_H$. Therefore, if the desired vertical reaction force is obtained successfully in accordance with this controller, the robot can hop the desired jumping height z_H stably.

D. Controller to absorb the impulse at touchdown

Let us assume that the robot touches down at $z = {}^d z$ and the velocity of COM at this moment is denoted by v_p. Generally speaking, a production of a impulsive shock on touchdown is responsible for a drastic change of COM acceleration at the moment of landing on the ground. Since the acceleration of COM just before touchdown is $\ddot{z} = -g$, we can design a controller to smooth the acceleration of COM between the aerial phase and the supporting phase. Additionally, in order to jump after landing, the controller have to transform seamlessly to the hopping controller described in the previous subsection. On that account, we will use the same structure as the control input (7), and consider the condition of the control parameter γ to satisfy $\ddot{z} = -g$ when $z = {}^d z$ and $\dot{z} = v_p$. Assuming that the acceleration of COM $\ddot{z}$ matches $-g$ when $z = {}^d z$ and $\dot{z} = v_p$, the following equation holds:

$$-\xi(q_1 + q_2)\left\{1 - \exp\left(1 - \frac{\gamma v_p^2}{\xi^2 q_1 q_2}\right)\right\} v_p = -g, \tag{13}$$

where $\rho = 1$ was given. Solving this for γ leads to

$$\gamma = \frac{\xi^2 q_1 q_2}{v_p^2}\left[1 - \log\left\{1 - \frac{g}{\xi(q_1 + q_2)v_p}\right\}\right] \equiv \gamma_p. \tag{14}$$

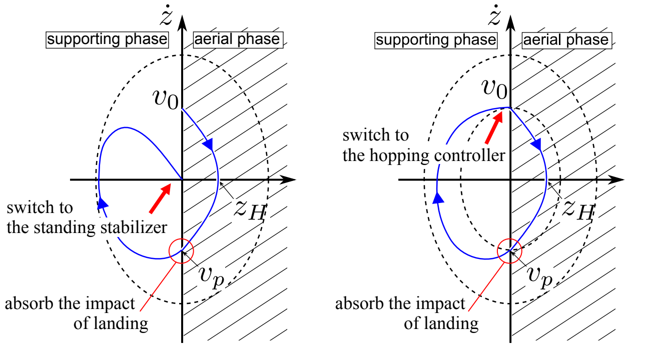
Therefore, the controller to absorb the shock impact on touchdown is as

$$f_z = \begin{cases} \tilde{f}_z^{\mathrm{III}} & (\mathrm{S}^{\mathrm{III}}1: \ \tilde{f}_z^{\mathrm{III}} > 0 \quad \text{and} \quad z \leq {}^d z) \\ 0 & (\mathrm{S}^{\mathrm{III}}2: \ \tilde{f}_z^{\mathrm{III}} \leq 0 \quad \text{or} \quad z > {}^d z) \end{cases} \tag{15}$$

$$\tilde{f}_z^{\mathrm{III}} = -m\xi^2 q_1 q_2 (z - {}^d z) - m\xi(q_1 + q_2)c^{\mathrm{III}}(z, \dot{z})\dot{z} + mg \tag{16}$$

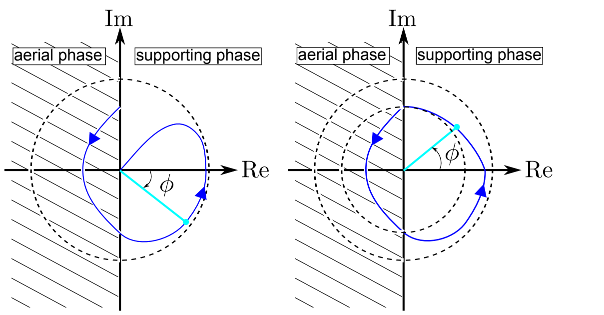
$$c^{\mathrm{III}}(z, \dot{z}) \equiv 1 - \rho \exp\left[1 - \gamma_p\left\{(z - {}^d z)^2 + \frac{\dot{z}^2}{\xi^2 q_1 q_2}\right\}\right]. \tag{17}$$

When Eq. (15) is provided as the control input, the dynamics of COM expressed as the following piecewise autonomous



(a) Standing control (b) Hopping control

Fig. 6. Trajectories of COM in the phase space. In the standing control as well as the hopping control, the controllers need to be designed to absorb the impact of landing.



(a) Standing control (b) Hopping control

Fig. 7. Trajectories of the complex number p_G on the complex plane. ϕ is the argument of p_G. α is the angle between the real axis and the direction of the eigenvector.

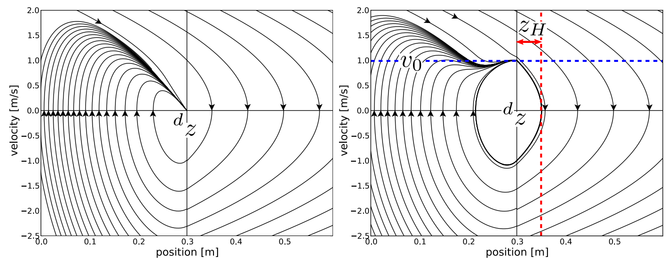
system:

$$\ddot{z} = \begin{cases} -\xi(q_1+q_2)c^{\mathrm{III}}(z,\dot{z})\dot{z} - \xi^2 q_1 q_2 (z - {}^d z) & (\mathrm{S^{III}1}) \\ -g & (\mathrm{S^{III}2}) \end{cases} \tag{18}$$

Fig. 5(c) shows the portions of solution curves of Eq. (18) for ${}^d z = 0.3,\ z_H = 0.05\ q_1 = 2.0,\ q_2 = 3.0$ and $\rho = 1.0$. As can be seen in the figure, solution curves continues smoothly when the robot transits from the flight phase to the supporting phase at $z = {}^d z$ and the impulsive shocks on touchdown is suppressed.

III. Transition of Controllers based on Command and State of Robot

Controllers (I), (II) and (III) defined by Eqs. (4), (7) and (15), respectively, can be interchanged one another by setting appropriate values to ρ and γ. Specifically, it equates to the controller (I) with $\rho = 0$ and $\gamma = 0$, the controller (II) with $\rho = 1$ and $\gamma = \gamma_0$ and the controller (III) with $\rho = 1$ and $\gamma = \gamma_p$. In order to design a unified controller which enables the robot to switch freely between standing and repetitive hopping, it is needed to build a global control scheme to select controllers (I), (II) and (III) depending on



(a) Landing to standing (b) Landing to hopping

Fig. 8. Phase portraits of the system. (a): Solid line represents the direction of the eigenvector of the regulator. (b): Horizontal dashed line represents v_0. Vertical dashed line represent the desired hopping height z_H.

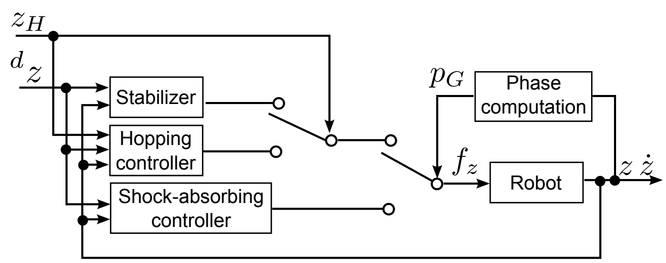


Fig. 9. A block diagram of the controller enabling transition of standing, hopping and shock-absorbing

the commanded value and the state of the robot. Our strategy is as follows:

- When the robot is in the supporting phase,
  - in the case of $z_H = 0$, switch to the controller (I) i.e. set $\rho = 0$ and $\gamma = 0$.
  - in the case of $z_H > 0$, switch to the controller (II) i.e. set $\rho = 1$ and $\gamma = \gamma_0$
- When the robot is in the aerial phase,
  switch to the controller (III) i.e. set $\rho = 1$ and $\gamma = \gamma_p$.

In the case where one expects the robot to stabilize from a flight phase, COM state on the phase plane should describe a trajectory like the blue solid line in Fig. 6(a). In order to construct the dynamical system which describes such a trajectory, the controller should correspond to the controller (III) before touchdown and switch to the controller (I) before stabilizing. Likewise, in the case where one expects the robot to hop repetitively, COM state on the phase plane should describe a trajectory like Fig. 6(b). In order to construct such a dynamical system, the controller should coincide with the controller (III) before touchdown in the same way and switch to the controller (II) before lifting off. Controller switching of this kind requires to abstract the phase information from the movement of COM. The following complex number p_G is available for circularity-free definition of the phase:

$$p_G = -(z - {}^d z) + \frac{\dot{z}}{\xi\sqrt{q_1 q_2}} i, \tag{19}$$

where i is the imaginary number. The minus sign of the real part and the scaling factor of the imaginary part are adjusted so that p_G moves in a clockwise circle on the complex plane. The complex number p_G describes a trajectory like Fig. 7(a) when COM follows Fig. 6(a), and describes like Fig. 7(b) when COM follows Fig. 6(b). Let us denote the argument of the complex number p_G as ϕ so as to consider the correspondence between the movement of p_G and the motion of the robot. When $\phi = -\frac{\pi}{2}$, p_G locates the boundary of the flight phase and the supporting phase and this means that the robot touches down. In a similar manner, when $\phi = 0$ the robot crouches down the most, when $\phi = \frac{\pi}{2}$ the robot lifts off the ground and when $\phi = \pi$ the robot reaches the apex in flight.

Fig. 9 exhibits a block diagram of the controller. In this figure, stabilizer, hopping controller and shock-absorbing controller mean the controllers (I), (II) and (III), respectively. The unified controller is expressed as following:

$$f_z = \begin{cases} \tilde{f}_z & (\text{S1}: \ \tilde{f}_z > 0 \quad \text{and} \quad z \leq {}^d z) \\ 0 & (\text{S2}: \ \tilde{f}_z \leq 0 \quad \text{or} \quad z > {}^d z) \end{cases} \tag{20}$$

$$\tilde{f}_z = -m\xi^2 q_1 q_2 (z - {}^d z) - m\xi(q_1 + q_2)c(z, \dot{z})\dot{z} + mg \tag{21}$$

$$c(z, \dot{z}) \equiv 1 - \rho(\phi) \exp\left[1 - \gamma(\phi)\left\{(z - {}^d z)^2 + \frac{\dot{z}^2}{\xi^2 q_1 q_2}\right\}\right]. \tag{22}$$

The control parameters ρ and γ, which are functionalized by ϕ, are defined as follows:

- If $z_H = 0$ is given as the commanded value,

$$\rho(\phi) = \begin{cases} 1 & (\phi > 0) \\ 0 & (\phi \leq 0) \end{cases}, \quad \gamma = \gamma_p. \tag{23}$$

- If $z_H > 0$ is given as the commanded value,

$$\rho = 1, \quad \gamma(\phi) = \begin{cases} \gamma_p & (\phi < 0) \\ \gamma_0 & (0 < \phi) \end{cases}. \tag{24}$$

Fig. 8(a) and Fig.8(b) show phase portraits for ${}^d z = 0.3$[m], $q_1 = 2.0$ and $q_2 = 3.0$, using Eqs. (20), (23) and (24). Fig. 8(a) is given $z_H = 0.0$[m], and Fig. 8(b) is given $z_H = 0.05$[m]. Either of the cases illustrates that the impulsive shock at touchdown is absorbed and the stabilization or the repetitive hopping from the flight phase is successfully achieved.

IV. Simulations

Simulations of repetitive hopping motions were executed to validate the proposed controller. A miniature humanoid robot "mighty" [17] was supposed. In the robot model, the total mass is concentrated at COM for simplicity. When it stands upright, the height of COM is 0.3[m]. The total weight is about 6.5[kg]. In all of the simulations described below, the initial condition was set for $(z, \ \dot{z}) = (0.3, 0.0)$, ${}^d z = 0.3$, $q_1 = 2.0$, $q_2 = 3.0$, $\rho = 0$ and $\gamma = 0$. The motion of COM is supposed to be confined to vertical direction only, and the whole body motion is generated by solving inverse kinematics from COM and feet positions.

The following three repetitive hopping motions were supposed:

(A) The desired hopping height is initially set to zero and changed to a constant height then backed to zero again.
(B) The desired hopping height is changed randomly during hopping.
(C) External forces are exerted during hopping.

Fig. 10 shows snapshots of the simulation (A). The others are omitted because of space limitations.

Fig. 11 shows loci of position, velocity and acceleration of COM in the simulation (A). We can observe that the robot successfully transit from stationary standing to hopping and back to standing. From Fig. 11(a) we can also see that the velocity of COM converges to v_0 at the moment that the robot lifts off, and that results in successful repetitive hoppings.

Fig. 12 shows loci of position, velocity and acceleration of COM in the simulation (B). Although the desired hopping height was changed completely at random, the robot, nevertheless, can continue hoppings and follow the changed height.

Fig. 13 shows loci of position, velocity and acceleration of COM and external forces in the simulation (C). Despite the exertion of external forces, the robot can suppress the drastic change of the acceleration at touchdown, and successfully absorb the impacts and achieve hoppings.

V. Conclusion

A novel repetitive hopping controller for biped robots based on the mass-concentrated model was proposed. This controller enabled a robot to transit between standing and repetitive hopping freely. In discontinuous variant constraint systems like hopping motion, it is challenging to continue a constant height hopping and absorb an impulsive shock at touchdown simultaneously. This difficulty was solved by designing three controllers that guarantee the global stability, namely, (I) the stabilizer, (II) the hopping controller, and (III) the shock-absorbing controller. Beside these, a global controller to transit these controllers depending on a commanded value and the state of the robot was designed. By combining these controllers, the robot can transit between standing and repetitive hopping freely and change its hopping height at will even in subjection to external forces. Additionally, this controller has advantage that it does not need a complicated parameter tuning.

Acknowledgment

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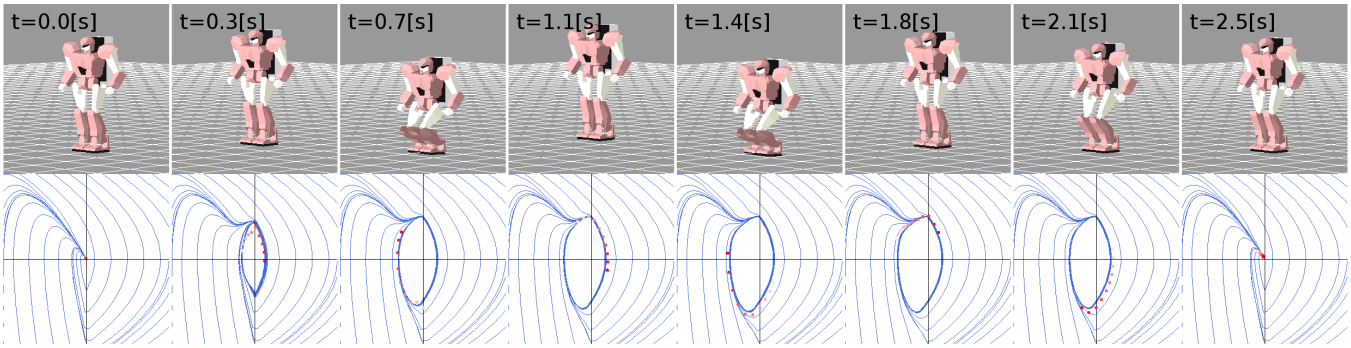
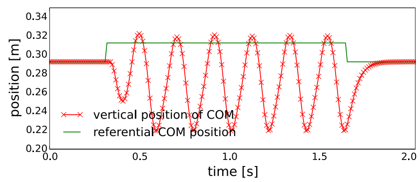
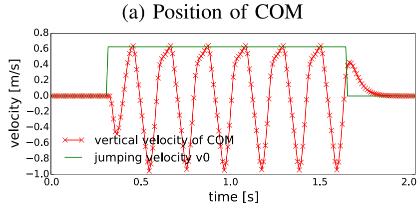


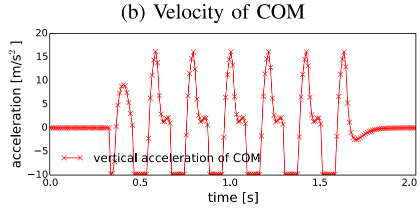
Fig. 10. Snapshots of the simulation



(a) Position of COM

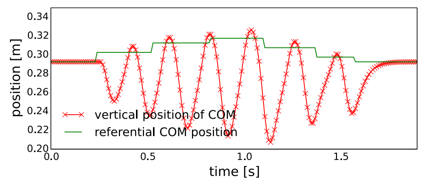


(b) Velocity of COM

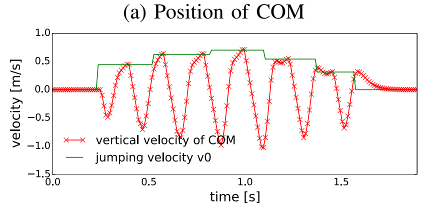


(c) Acceleration of COM

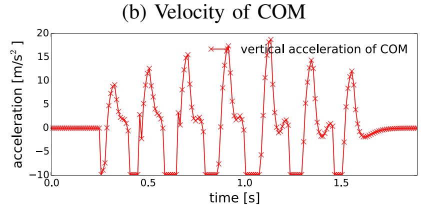
Fig. 11. Results of the simulation. The desired hopping height was set to $z_H = 0.02$[m].



(a) Position of COM

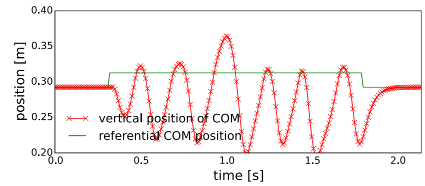


(b) Velocity of COM

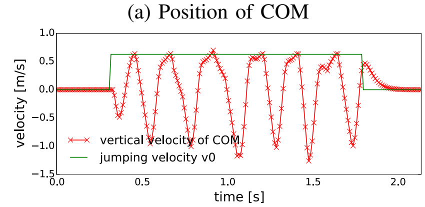


(c) Acceleration of COM

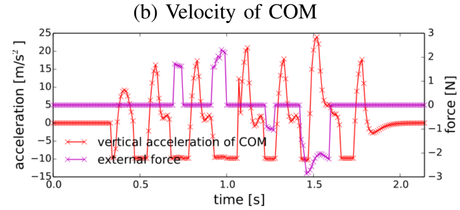
Fig. 12. Results of the simulation. The desired hopping height was randomly set to $z_H = 0.0 \sim 0.06$[m].



(a) Position of COM



(b) Velocity of COM



(c) Acceleration of COM

Fig. 13. Results of the simulation. The desired hopping height was set to $z_H = 0.02$[m].

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