

Reflexive Step-out Control Superposed on Standing Stabilization of Biped Robots

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Abstract—A novel step-out controller which is superposed on the stationary standing controller and enables an automatic transition to such a reflexive behavior in emergency cases is proposed in the framework of the dynamics morphing. The step-out motion has a distinctive property in spite of the simpleness of the motion itself, which requires temporarily inconsistent escapement from an equilibrium point on purpose and transition to another stable equilibrium point produced by the enlargement of the supporting region. It is largely different from steady motions such as standing and periodic stepping, which keep stability around a fixed point. In our scheme, the position of ZMP is determined based on a linear state feedback in the stationary phase, while it is forced to be positioned in the pivot sole to emerge instantaneous step-out motion in the emergency phase. Immediately after the landing, it returns to be manipulated to stabilize the robot again. As the result, two requirements which tend to conflict with each other, namely, the stabilization of COM and the dynamically feasible manipulation of reaction forces, are concurrently achieved. Our scheme is based on the maximally stable COM-ZMP regulator, which enables one to judge if the step-out is necessary for fall-prevention, so that a complete controller on which the stabilization and the emergent disequilibrating maneuver are compatible was implemented.

Index Terms—Biped control, Reflexive step-out, Voluntary stepping, ZMP manipulation

I. INTRODUCTION

Bipedalism in general includes a variety of motion such as standing stabilization, step-out for fall-prevention, walking, running, jumping, etc. While those motions share a common principle that they are achieved through discontinuous deformation of the supporting region by stepping and manipulation of the reaction force within the supporting region, the dynamics of those motions have quite different structures and characteristics from each other. Particularly, the step-out motion, which this paper deals with, has a distinctive property where a subject temporarily disequilibrates itself on purpose and transits to another point of equilibrium. It is largely different from standing and periodic stepping motions, which keep stability around a fixed point, so that it requires a complicated control in contrast with the simpleness of the motion itself.

The conventional researches on biped motions have not discussed the above issue sufficiently since many of them only targeted stable and steady motions. Humanoid robots which can perform various tasks and can cope with sudden perturbations by step-out have relied on a combination of

carefully designed motion trajectories and heuristic controls [1], [2], [3], [4], [5], [6], [7], [8], [9], so that the methodology of how to unify such regular and irregular motion controllers hasn't been clarified.

Sugihara[10] discussed a design of a standing controller which maximizes the performance of the standing stability under a given supporting region, and simultaneously, a maneuver to choose the landing point for the fall-prevention when the robot goes into an unstabilizable state. However, the problem of step-out control to carry the foot to the desired landing point over the above mentioned paradoxical problem has not been solved.

This paper proposes a novel step-out controller which is superposed on the stationary standing controller and enables an automatic transition to such a reflexive behavior in emergency cases in the framework of the dynamics morphing[11]. In the stationary phase, the position of ZMP[12], which works as the input to the system, is determined based on a linear state feedback of the center of mass. On the other hand, in the emergency phase, ZMP is forced to be positioned in the pivot sole to emerge an instantaneous step-out motion, and immediately after the landing, it returns to be manipulated to stabilize the robot again by introducing an artificial decaying dynamics to the offset of ZMP caused by the forced jump. As the result, two requirements which tend to conflict with each other, namely, the stabilization of COM and the dynamically feasible manipulation of reaction forces, are concurrently achieved.

II. BIPEDAL MOTION CONTROL BASED ON DYNAMICS MORPHING[13]

The dynamics morphing is a framework to design a controller which enables seamless transitions between various motions by continuously morphing the dynamical structure of the feedback system as the whole. The dynamical structures of each motion are designed based on the dynamical constraint and the motion context. Here, we are targeting a design of a biped robot controller in the lateral plane upon this framework.

Let us assign x and z axes along with the lateral and vertical directions, respectively, as depicted in Fig.1, and denote the location of the center of mass (COM) and the zero-moment point (ZMP)[12] by $\mathbf{p} = [x \ z]^T$ and $\mathbf{p}_Z = [x_Z \ z_Z]^T$, respectively. Suppose the inertial torque about COM of the

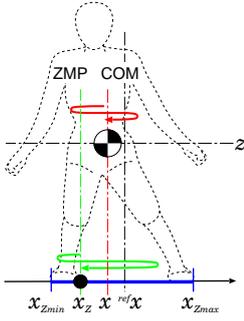


Fig. 1. COM-ZMP model of a biped system in lateral plane

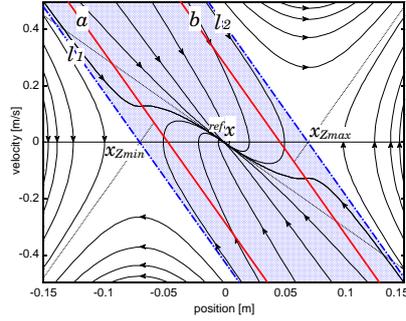


Fig. 2. A phase portrait of the maximally stable COM-ZMP regulator. The area of standing-stable region is maximized.

robot is sufficiently small to be ignored and the height of COM z is almost constant for simplicity, we get the following simplified equation of motion[14]:

$$\ddot{x} = \omega^2(x - x_Z), \quad (1)$$

where $\omega \equiv \sqrt{g/z}$ and $g = 9.8[\text{m/s}^2]$ is the acceleration due to the gravity. An important constraint that ZMP p_Z has to lie within the supporting region is posed as

$$x_{Z\min} \leq x_Z \leq x_{Z\max}, \quad (2)$$

where $x_{Z\min}$ and $x_{Z\max}$ are the right and left boundaries of the supporting region in x -axis, respectively. Even on this simplest dynamics model, the constraint (2) makes the control problem challenging. Here, we define the referential ZMP as the input to the system with respect to the referential COM ${}^d x$ as follows:

$$\tilde{x}_Z = {}^d x + (q+1) \left(x - {}^d x + f(\zeta) \frac{\dot{x}}{\omega} \right) \quad (3)$$

$$x_Z = \begin{cases} x_{Z\max} & (\text{S1} : \tilde{x}_Z > x_{Z\max}) \\ \tilde{x}_Z & (\text{S2} : x_{Z\min} \leq \tilde{x}_Z \leq x_{Z\max}) \\ x_{Z\min} & (\text{S3} : \tilde{x}_Z < x_{Z\min}) \end{cases}, \quad (4)$$

where

$$f(\zeta) \equiv 1 - \rho \exp \left\{ 1 - \frac{(q+1)^2 \zeta^2}{r^2} \right\} \quad (5)$$

$$\zeta \equiv \sqrt{x^2 + \frac{\dot{x}^2}{\omega^2 q}}, \quad (6)$$

q , r and ρ (constant values) are the control parameters to be designed. Then, the behavior of COM is ruled by the following piecewise system:

$$\ddot{x} = \begin{cases} \omega^2 x - \omega^2 x_{Z\max} & (\text{S1}) \\ -\omega(q+1)f(\zeta)\dot{x} - \omega^2 q(x - {}^d x) & (\text{S2}) \\ \omega^2 x - \omega^2 x_{Z\min} & (\text{S3}) \end{cases}. \quad (7)$$

This system coincides with *the maximally stable COM-ZMP regulator*[10] for $\rho = 0$, the phase portrait of which (with $q = 0.5$) is plotted as Fig.2 shows. The blue region in the figure is the set of initial states which stably converge to the reference, so that we call it *the standing-stable region*. The

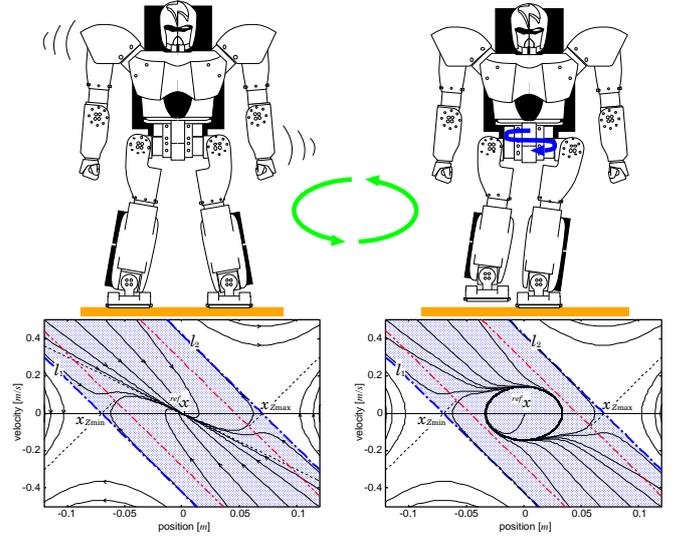


Fig. 3. Dynamics morphing to synthesize biped motions

system has a stable limit cycle represented by the following equation in (S2) if $\rho > e^{-1}$:

$$(x - {}^d x)^2 + \frac{\dot{x}^2}{\omega^2 q} = \frac{(1 + \log \rho)r^2}{(q+1)^2}. \quad (8)$$

Particularly when $\rho = 1$, it becomes a harmonic oscillation with the amplitude $\frac{r}{q+1}$ and the period $\frac{2\pi}{\omega\sqrt{q}}$.

In addition, Sugihara[13] proposed a controller by which the stepping is synchronized to the oscillation of ZMP so as to automatically satisfy the dynamical constraint about the unilaterality of reaction forces. As the result, a seamless control of standing and stepping motions is achieved as depicted by Fig.3. He[15] also developed a longitudinal walking controller to follow an arbitrary referential velocity given at random timing and to cope with unexpected external forces.

III. SUPERPOSING REFLEXIVE STEP-OUT ON STANDING CONTROL

A. Dynamics of Stepping

Whether the COM at a certain state (a pair of position and velocity) is stabilizable without varying the supporting region or not can be judged when the standing controller is designed as the maximally stable COM-ZMP regulator[10], since the standing-stable region is defined only by two lines. Namely, COM is stable on an invariant stance if the following condition is satisfied:

$$x_{Z\min} < x + \frac{\dot{x}}{\omega} < x_{Z\max}. \quad (9)$$

Furthermore, if the state of COM doesn't satisfy the above condition (9), a foot location to land in the future for a recovery of the stability is determined as follows:

(SL: if $x + \frac{\dot{x}}{\omega} \geq x_{Z\max}$)

$${}^d x_L = {}^d x + (q+1) \left(x - {}^d x + \frac{\dot{x}}{\omega} \right), \quad {}^d x_R: \text{fixed} \quad (10)$$

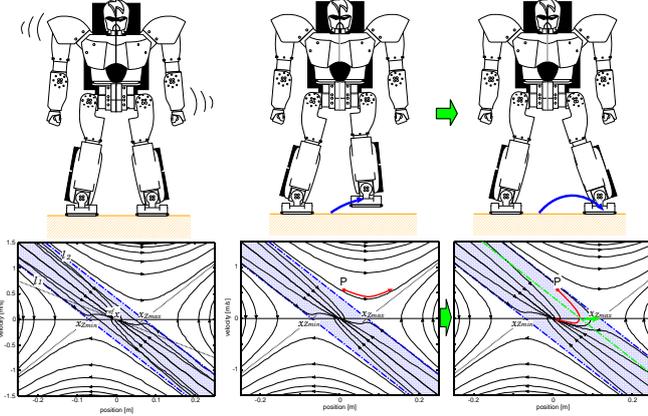


Fig. 4. Step-out motion and deformation of the supporting region

$$(SR: \text{if } x + \frac{\dot{x}}{\omega} \leq x_{Zmin})$$

$${}^d x_L: \text{fixed}, \quad {}^d x_R = {}^d x + (q+1) \left(x - {}^d x + \frac{\dot{x}}{\omega} \right) \quad (11)$$

where ${}^d x_L$ and ${}^d x_R$ are the desired landing positions of the left foot and the right foot, respectively, in order to regain stability. This maneuver is based on *the strongly standing-stable condition* [10] and more preferable than that based on the capture point [7] from the viewpoint of controllability immediately after landing.

Even though it finds where to land for fall-avoidance, however, the step-out control to carry the foot to the desired position is not easy. Let us consider a situation where the state of COM comes into (SL) as depicted in Fig.4. COM comes back to be stable if the robot steps out the left foot based on Eq.(10). On the other hand, the controller defined by Eq.(4) works to leave ZMP at $x_Z = x_{Zmax}$ and to hold back COM at the maximum acceleration on the current supporting region. In order to step out the left foot, the robot once has to shift ZMP to the pivot foot, namely, the right foot, so as to get the reaction force exerted to the left sole to be zero. As the result, COM is accelerated to fall more to the left side. This problem hasn't been thoroughly discussed, despite it reveals a paradox that a motion which people unconsciously do without any difficulties requires an inconsistent manipulation of ZMP temporarily with the intended purpose.

B. Forced positioning of ZMP to recover stability

As explained in the previous subsection, ZMP is required to be shifted into the right sole in order to lift up the left foot when the state of COM is in (SL), and vice versa, while it has to be manipulated based on the controller (4) again as soon as the supporting region is enlarged by step-out. This is implemented by redefining ZMP as the input by the following equation:

$$x_Z = \bar{x}_Z + \Delta x_Z, \quad (12)$$

where \bar{x}_Z is the position of ZMP determined by Eq.(4), and Δx_Z is a parameter which is enforced to be $x_R - \bar{x}_Z$ at (SL)

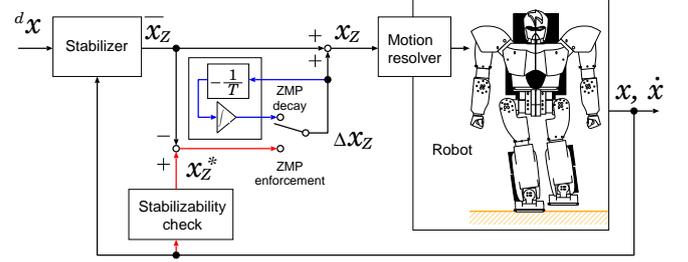
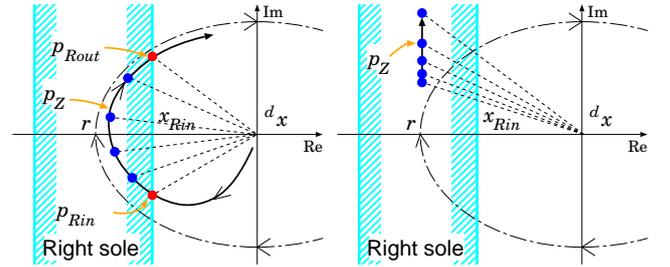


Fig. 5. A block diagram of a controller enabling forced positioning of ZMP which switches ZMP enforcement and ZMP decay



(a) When converging to the limit cycle, the locus of ZMP is segmented by the inner edge of the pivot sole. (b) When ZMP is enforced on the pivot, the locus of ZMP doesn't intersect with the inner edge of the pivot sole.

Fig. 6. Locus of ZMP phase in the pivot sole

and $x_L - \bar{x}_Z$ at (SR), and otherwise, is changed according to the following differential equation:

$$\Delta \dot{x}_Z = -\frac{\Delta x_Z}{T}, \quad (13)$$

where T is a time constant of the decay of Δx_Z . This decaying dynamics of Δx_Z works to automatically compensate the forced jump of ZMP and smoothly go back to the stabilization control. Fig.5 shows a block diagram of the above system.

C. Foot-lifting control

The state transition is caused by the touch-down and detachment-off accompanying the foot-lifting. If the foot configuration satisfies Eq.(10) or Eq.(11) at the touch-down, the state shifts to (S2) from (SL) or (SR), respectively.

Let us review the previous foot-lifting control during a stationary alternate stepping [13]. First, a complex number p_Z is defined from the position of ZMP and the velocity of COM to represent the system phase as follows:

$$p_Z \equiv x_Z - {}^d x - \frac{(q+1)\dot{x}}{\omega\sqrt{q}} i, \quad (14)$$

where i is the unit imaginary number. Now, we assume that the right foot is the pivot foot. When x_Z moves within the contacting area in the right sole, the locus of p_Z is segmented at two intersection points with the inner edge of the right sole x_{Rin} as depicted in Fig.6(a). Here, we note those two points by p_{Rin} and p_{Rout} , namely,

$$\bar{p}_{Rin} = x_{Rin} - \sqrt{|p_Z|^2 - x_{Rin}^2} i \quad (15)$$

$$\bar{p}_{Rout} = x_{Rin} + \sqrt{|p_Z|^2 - x_{Rin}^2} i. \quad (16)$$

Based on this fact, a relative assignment of p_Z to the segmented portion is successively estimated, and the lifting phase of the left foot ϕ_L is defined as

$$\phi_L \equiv \frac{\angle(p_Z/\bar{p}_{Rin})}{\angle(\bar{p}_{Rout}/\bar{p}_{Rin})}, \quad (17)$$

It is guaranteed that ZMP lies within the left sole as long as p_Z satisfies

$$0 < \phi_L < 1. \quad (18)$$

Then, when a flat terrain is assumed, the lifting height of the left foot z_L with respect to ϕ_L is defined as follows:

$$z_L = \frac{1}{2} \frac{h|p_Z|}{r} \sigma_1(\rho) (1 - \cos 2\pi\phi_L), \quad (19)$$

where h is a constant, and $\sigma_1(\rho)$ is defined as follows:

$$\sigma_1(\rho) \equiv \begin{cases} 1 & (\rho > 1) \\ \frac{\rho - e^{-1}}{1 - e^{-1}} & (e^{-1} \leq \rho \leq 1) \\ 0 & (0 \leq \rho < e^{-1}) \end{cases}. \quad (20)$$

In the case that the left foot is the pivot foot, ϕ_R and accordingly the lifting height of the right foot z_R are also defined as well.

The problem of the above method is that the stepping foot does never touch down to the ground if ZMP is forced to be positioned in the pivot sole by the method described in the previous subsection since the locus of p_Z and the inner edge of the pivot sole doesn't intersect as depicted in Fig.6(b). In the meantime, the velocity of COM increases and the desired landing position defined by Eq.(10) and (11) diverges outward.

In order to resolve this problem, let us redefine the foot-lifting height instead of Eq.(19) as follows:

$$z_* = \frac{1}{2} \frac{h|p_Z|}{r} \sigma_1(\rho) \sigma_2(x) (1 - \cos 2\pi\phi_*), \quad (21)$$

where $\sigma_2(x)$ is defined as

$$\sigma_2(x) \equiv \begin{cases} \frac{|x_K - x_P|}{s_{\max}} & (|x_K - x_P| < s_{\max}) \\ 0 & (|x_K - x_P| \geq s_{\max}) \end{cases}. \quad (22)$$

s_{\max} is the maximum step width which is kinematically accepted. K and P are for L and R , respectively, when the right foot works as the pivot foot, and for R and L , respectively, vice versa. It inhibits the foot-lifting height with respect to the distance of the stepping foot from the pivot foot, and avoids the divergence of the desired landing position. In addition, ρ is automatically set for 1 when ZMP is enforced within the pivot sole in order to emerge the foot-lifting.

On the other hand, the sideward positions of the stepping foot x_L and x_R are defined only in the detachment phase in order to follow the desired positions given by Eq.(10) and (11) as follows:

$$\ddot{x}_L = k(d x_L - x_L) - c\dot{x}_L \quad (23)$$

$$\ddot{x}_R = k(d x_R - x_R) - c\dot{x}_R. \quad (24)$$

It is based on the similar idea to Sugihara[16]. In the supporting phase, they are controlled to stay at the same positions.

IV. SIMULATIONS

We conducted some simulations to verify the proposed method, supposing an anthropomorphic robot mighty[17]. In the robot model, the total mass was concentrated at COM for simplicity, $z = 0.25$ [m], $h = 0.02$ [m] and $q = 0.5$. The positions of the left and right feet were represented by their center points, where the widths of feet were both 0.07[m]. The initial distance between the left and right feet was set for 0.084[m]. The other control parameters were $T = 0.1$, $s_{\max} = 0.3$, $k = 5 \times 10^4$ and $c = 2 \times 10^2$. The following automatic adjustment of the referential point of COM to the center of both feet was applied in the detachment phase:

$$d_x = \frac{x_L + x_R}{2} \quad (25)$$

$$r = \frac{x_L - x_R}{2}. \quad (26)$$

Concerning with the foot landing, the perfect plastic collision was assumed and the foot instantaneously contacts to the ground without slip, oscillation and chattering.

The following two situations in which the step-out motion was emerged were supposed:

- (I) The referential point of COM is assigned outside of the supporting region.
- (II) The state of COM is forcibly moved to (SL) by an external force.

Fig.8 shows loci of COM, ZMP and both feet when the referential point of COM is moved outward from the outer edge of the left foot under the assumption of situation (I), where the supporting region is filled by dots. One can see that ZMP jumped to the center of the right foot at the time when the state of COM reached the boundary of the standing-stable region, and the step-out motion of the left foot was emerged due to the movement of \tilde{x}_Z during the process. After the landing, the supporting region was enlarged and COM came again into the standing-stable region. Eventually COM converged to the referential point, which was automatically adjusted to the midpoint of the both feet. Snapshots of the robot motion and the morphing phase portrait of COM are shown in Fig.7.

Fig.10 shows loci of COM, ZMP, both feet and the external force applied to COM under the assumption of situation (II). A history of the external force normalized by the weight of the robot is plotted in the bottom graph. Although the step-out motion was emerged by different cause from the one in (I), it showed similar behaviors of ZMP and the left foot accompanying the movement of \tilde{x}_Z to that of (I) Snapshots of the robot motion and the morphing phase portrait of COM are shown in Fig.7.

V. CONCLUSION

A novel step-out controller for biped robots including humanoid robots in emergency cases was proposed. First,

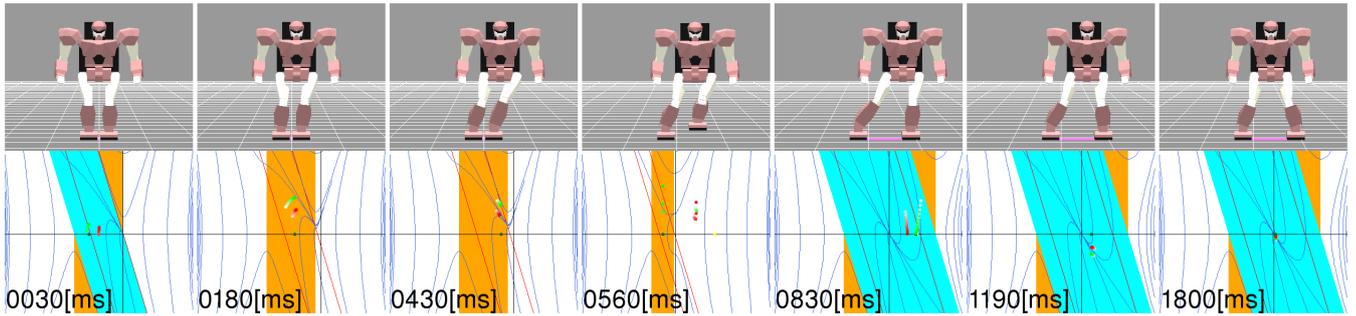


Fig. 7. Snapshots of a step-out motion at will

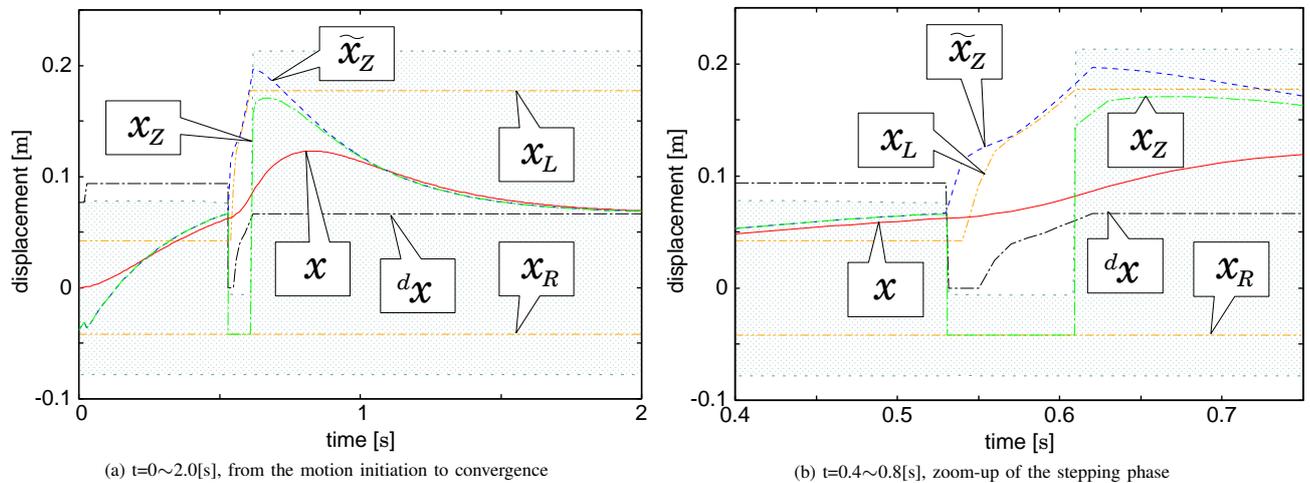


Fig. 8. Loci of COM, ZMP and feet in a step-out motion at will

the author pointed out the difficulty lying on the step-out motion in which a temporarily inconsistent manipulation of ZMP with the intended purpose is required. Then, a technique to superpose such an emergent controller on the steady stabilization controller was discussed, where ZMP is forced to be positioned in the pivot sole for instantaneous step-out and it goes back to be manipulated for stabilization of COM after the landing through the compensation of the jump of ZMP achieved by an artificial decaying element.

The proposed scheme is based on the maximally stable COM-ZMP regulator, which enables one to judge if the step-out is necessary for fall-prevention, so that a complete controller on which the stabilization and the emergent disequilibrating maneuver are compatible was implemented.

As the simulated examples show, the proposed controller works to regain the stability when a robot loses it for whatever reason. This is beneficial for a design of a simple control architecture to unify various biped motions. The author believes that it will be the basis of highly autonomous humanoid robots.

The proposed control only supposes cases that a robot can prevent from falling by one step. When stronger perturbations are applied, multiple steps might be required. Obviously, in such cases, a robot has to cross its legs in order to

avoid self-collision, and the model confined in the lateral plane doesn't work. We need to discuss it in the context of three-dimensional motions. Another issue to be discussed in the future is the landing control in order to guarantee firm contacts in quite a short time even on irregular terrains. Particularly, a more adaptive control of the vertical motion of the foot will be desired.

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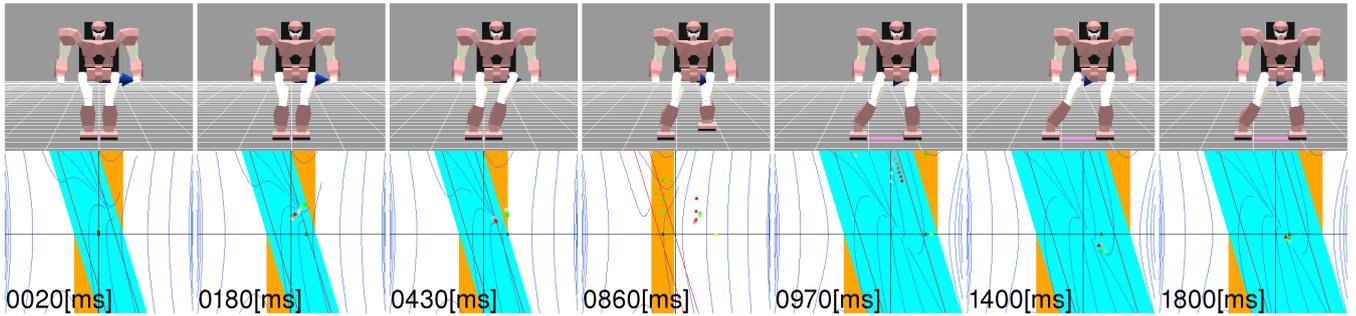


Fig. 9. Snapshots of a step-out motion in an emergency case

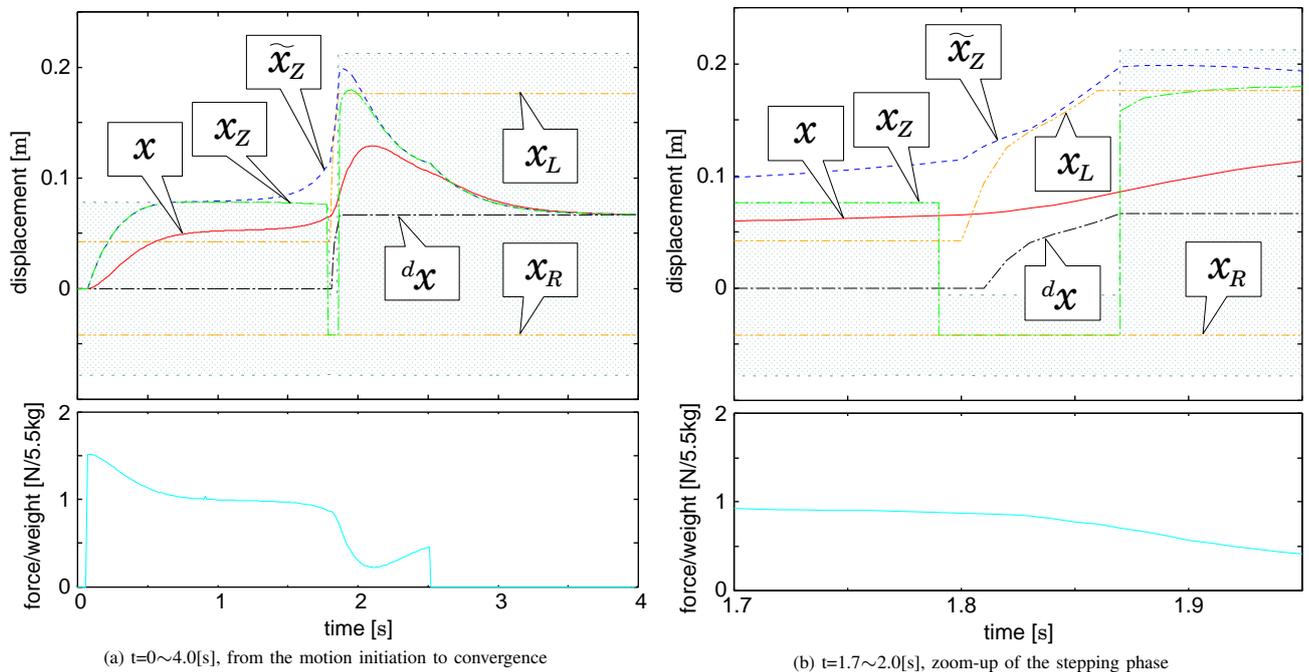


Fig. 10. Loci of COM, ZMP and feet in a step-out motion due to an emergency

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